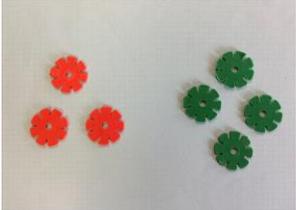
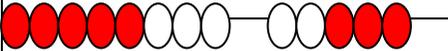


GLEN HILLS CALCULATION POLICY

PROGRESSION IN ADDITION

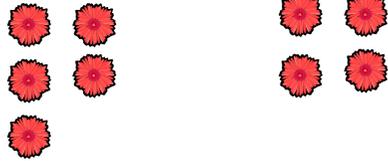
Children should learn that addition is increasing a number or quantity and adding two or more numbers or quantities together

Children should carry out addition using objects and practical apparatus before and alongside representations using numbers and symbols

Steps	Examples	Suggested activities	Models and images	Vocabulary
4+	<p>Adding one or more to a given number of objects.</p>  <p>Adding two or more quantities together.</p> 	<p>Singing songs to count up and back. Recognising numbers before and after if missing on a number line. Ordering number tiles. Songs, rhymes, games, counting on and back... using objects and apparatus (eg 'One, two, three, four five, once I caught a fish alive....' 'One, two, buckle my shoe...') Playing games ("You are on number six and the dice shows 3. What number will you land on next?")</p> <p>Children should learn and quickly recall number bonds within 10.</p>	<p>bead strings</p>  <p>fingers,</p>  <p>cubes, counters</p> <p>number lines</p> <p>practical objects</p> 	<p>add, more, and, make, sum, total, altogether, score, double, one more, two more, ten more, is the same as, equals, answer, count, sort, group,</p>

Step A

$5 + 4 =$



5 flowers and 4 flowers. How many flowers are there altogether?

$7 + 4 =$

7 people are on the bus. Four more get on at the next stop.

How many people are on the bus now?



$11 = 7 + 4$

Show children that calculations can be written in different ways

Children could draw a picture to help them work out the answer or use objects to help them (try counting cars or conkers).

Introduce the symbols for equals and add
Children could use dots or tally marks to represent objects (quicker than drawing a picture).

Children continue to develop their recall of number bonds including doubles and halves.

Practical contexts of counting objects, fingers

- counters
- beads
- hoops trays

bead strings - 1 more 1 less



**pegs on a coat hanger
Cuisenaire**

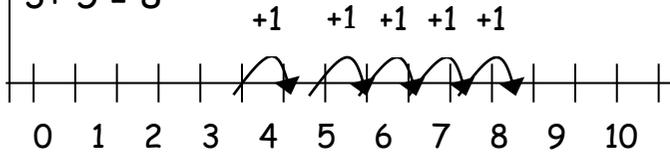


as above and also:

plus, near double, how much more is..? equals, sign, number sentence, operation, count on, near double, plus, equals, sign

Step B

$3 + 5 = 8$



$16 + 3 = 19$



Children begin to use a number line marked in ones to count up

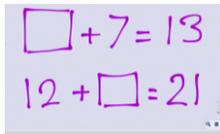
The children then extend this to drawing their own 'blank number line', starting at the chosen number.

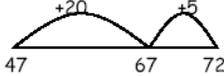
Link to developing mental recall of number facts.

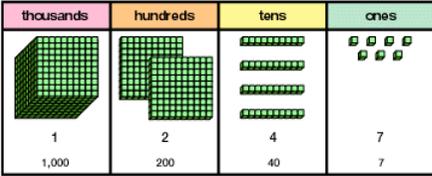
**Different range of number lines marked in 1's 2's 5's 10's
ITP 'Counting on and back'**

- Using beadstrings
- 100 square
- counting stick
- 100 square
- multi-link cubes
- Cuisenaire



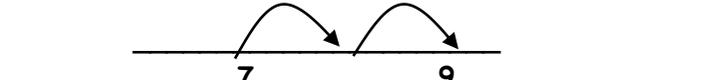
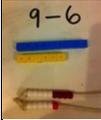
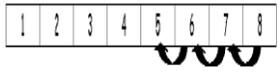
	<p>The children should learn that addition is commutative eg $5 + 7$ is the same as $7 + 5$ and also the inverse link between addition and subtraction. They should be able to use this knowledge to solve empty box sums.</p>		<p>100 square</p> 	
<p>Step C</p>	<p>As children become more confident with counting up to 100 they can begin to add multiples of 10 to a given number</p> <p>eg What is 10 more than 28? What is 30 more than 42?</p> <p>Formal recording at this stage will be horizontal</p> <p>eg $51 + 7 = 58$ or $62 + 20 = 82$</p> <p>Quick recall of number bonds within 10 and 20</p> <p>Eg: Adding a number in different ways up to 10</p> <p>How many different ways can you make a total of 7 $3+4=7$ $2+5=7$ $1+6=7$ $6+1=7$ etc</p> <p>Children know and recognise doubles, and can use doubles and near doubles</p> <p>$7+3 = 10$ and one more =11</p>	<p>Bead strings and 100 squares can be used to model adding multiples of 10 to any given number.</p> <p>100 squares and bead strings model counting in multiples of 10. Continue to develop mental recall of number facts up to and within 100. Relate to subtraction as the inverse of addition.</p> <p>At this stage children should have confident recall and clear mental image of numbers within 10 and 20 and their relationships (bead strings). They should be beginning to be confident when handling numbers up to 100.</p>	<p>straw bundles (base 10)</p> <p>blank number lines</p>  <p>number lines marked in tens</p>  <p>Cuisenaire</p>	
<p>Step D</p>	<p>The children should then move onto adding 2 two digit numbers eg</p> <p>My sunflower is 47cm tall. It grows another 25 cm. How tall is it now?</p> <p>They should use empty lines and then move onto partitioning as they become more confident with knowledge of place value.</p> <p>Partitioning:</p>	<p>Drawing an empty number line helps children to record the steps they have taken in their calculation (start at $47 + 20$ then add 5). This is more efficient than counting on in ones. Allow the children to choose the most appropriate jumps along the number line.</p>	<p>number lines</p> <p>beadstrings</p>  <p>multi-link.</p> 	<p><i>as above and also:</i></p> <p>addition , one hundred more, tens boundary, calculate, calculation, mental calculation, jotting,</p>

	$36 + 42$ $6 + 2 = 8$ $30 + 40 = 70$ $= 78$ $30 + 40 = 70$ $6 + 2 = 8$ $= 78$ <p>Collect the numbers together and recombine.</p>		 <p>Straw bundles</p>	symbol inverse												
Step E	<p>The children should then move onto a more formal recording method of partitioning that introduces the use of columns for the digits.</p> $40 + 7$ $+ \underline{20 + 5}$ $60 + 12 \quad \text{Recombine to make } 72$ <p>Continue to make sure that practical apparatus is used to demonstrate and help children to understand the process</p>	<p>Children use partitioning to add larger numbers, when children have a sound understanding of the value of the digits in a two digit number eg $47 = 40 + 7$</p> 	<p>Dienes base 10 equipment</p>  <p>place value chart</p> <table border="1" data-bbox="1429 799 1805 954"> <thead> <tr> <th>thousands</th> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>1 1,000</td> <td>2 200</td> <td>4 40</td> <td>7 7</td> </tr> </tbody> </table>	thousands	hundreds	tens	ones					1 1,000	2 200	4 40	7 7	
thousands	hundreds	tens	ones													
																
1 1,000	2 200	4 40	7 7													
Step F	<p>As children move into adding 3 digit numbers, the expanded vertical method becomes more practical.</p> $132 + 254$ 139 $+ \underline{254}$ $\underline{13}$ 80 $\underline{300}$ 393 <p>Remember to add units, then tens then hundreds. NOTE: Use this expanded method to model the carrying, leading into the compact method.</p>	<p>Children will be taught written methods for those calculations they cannot do 'in their heads'. Expanded methods build on mental methods and make the value of the digits clear to children.</p>	<p>Continue to use Dienes base 10 equipment, place value boards and place value counters</p>  <table border="1" data-bbox="1429 1214 1865 1385"> <thead> <tr> <th>thousands</th> <th>hundreds</th> <th>tens</th> <th>ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td>1 1,000</td> <td>2 200</td> <td>4 40</td> <td>7 7</td> </tr> </tbody> </table>	thousands	hundreds	tens	ones					1 1,000	2 200	4 40	7 7	<p><i>as above and also:</i></p> <p>hundreds boundary, method, equation, inverse, one hundred more, one hundred less,</p>
thousands	hundreds	tens	ones													
																
1 1,000	2 200	4 40	7 7													

<p>Step G</p>	<p>Compact vertical written method: $156 + 138$ The postman delivered 156 letters on Tuesday and 138 letters on Wednesday. How many letters did he deliver?</p> $\begin{array}{r} 156 \\ + 138 \\ \hline 294 \\ 1 \end{array}$	<p>When children are confident using the expanded method this can be 'squashed' into the traditional compact method where numbers may be 'carried' into the next column. Continue using practical apparatus until children are confident and understand each step. Always start adding the units first</p>	<p>place value chart</p> 	<p><u>as above and also:</u> increase inverse</p>
<p>Step H</p>	<p>Moving onto decimals and larger numbers: A parcel weighs 2.35 kg and another weighs 4.9 kg. What is their total weight?</p> $\begin{array}{r} 2.35 \\ + 4.90 \\ \hline 7.15 \end{array}$	<p>As children become more confident they will use traditional compact method using decimals, and for adding larger 4 and 5 digit numbers</p>		<p><u>as above and also:</u> units boundary tenths boundary , strategy, thousandth</p>

PROGRESSION IN SUBTRACTION

Children are taught that subtraction can be the process of taking one or more away from a given number or finding the difference between two numbers or quantities

Steps	Examples	Suggested activities	Models and images	Vocabulary
FS	<p>Finding one more or one less than a given number</p> <p>Taking one or more items from a given number</p> <p>Comparing different amounts eg How many more marbles do you have than me?</p>	<p>Songs, rhymes, games, counting on and back... using objects and apparatus (eg 'Five little speckled frogs'; 'Ten green bottles')</p> <p>Recognising numbers before and after if missing on a number line. Songs, rhymes, games, counting on and back in ones on a number line. Use number lines and tracks marked in ones</p> <p>Children should learn subtraction bonds to 5 eg $5 - 1 = 4$</p>	<p>bead strings fingers, cubes, counters comparing objects on a line practical objects in classroom setting number line, intervals of one number track</p>	<p>how many more to make..., how many more is ... than...? take (away), leave how many have gone? How many are left / left over? One less, two less..., ten less, . how many fewer is... than...?, .difference between, is the same as, is the same as</p>
Step A	<p>Counting <u>up</u> and <u>back</u> on a number line (marked in ones)</p> <p>Record steps <i>back, below</i> the number line $5 - 3 = 2$</p>  <p>Record steps <i>up, above</i> the number line, $9 - 7 = 2$</p> 	<p>Children by Year 1 need to experience subtraction as both difference and take away.</p> <p>Begin with prepared number lines and move onto children drawing their own.</p> <p>Children should learn number bonds to 10 and associated subtraction facts.</p> <p>Ensure that children are aware of the link between addition and subtraction and that they understand that subtraction can not be done in any order.</p>	<p>ITP 'difference' </p> <p>Bead string Bar model to show difference</p> <p>Number lines marked in ones Counters, comparing objects Cuisenaire</p>  <p>Number tracks </p>	<p><u>as above and also:</u></p> <p>subtract, take away, how much more is...?, minus, leave, how much less is...? is the same as, equals, sign, operation, number sentence</p>

Step B

8-3=

There are eight biscuits on this plate. Take three of the biscuits to eat. How many biscuits are left on the plate?



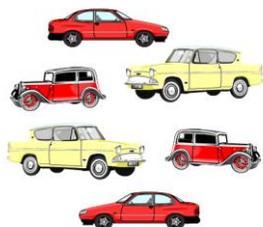
Take away

A teddy bear cost £5 and a doll cost £2. How much more does the bear cost?



Find the difference

*Here are six toy cars.
How many more cars are needed to make a set of eight cars?*



solve missing number problems e.g.

11 - □ = 8 □ = 13 - 2 3 = □ - □

Drawing a picture helps children to visualise the problem.

Children at this stage will also develop knowledge and use of number bonds both to and within 20 to perform calculations. They may be taking away objects, or counting back to 'take away' where differences are small.

Where differences between numbers are larger, children find the difference by counting on from the smallest number to the largest.

Use images / models / pictures and practical objects in context to support.

The use of the number line may be extended to using a 'blank number line' where children can start from any given number and count on

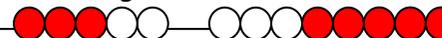
Children build on understanding of subtraction alongside its inverse (addition), in building mental strategies and understanding of number bonds up to and within 10 and 20.

They practice mental recall of number bonds both to and within 10 and 20 (linking addition and subtraction).

ITP 'difference'



Bead strings in tens to a hundred



Number lines intervals in ones

Number lines intervals of 10

ITP 'Number facts'

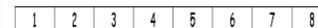
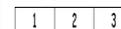
Blank number lines (starting at numbers other than 0)

Singapore Bar Model

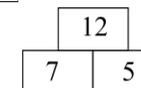
Cuisenaire



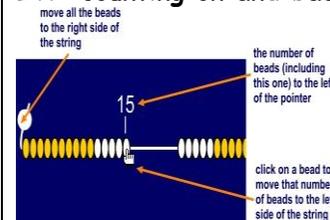
Number tracks to show the difference



Number trios



ITP 'counting on and back'

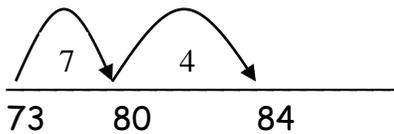


as above and also:

one hundred less, tens boundary, symbol, calculate, calculation, mental calculation, jotting, correct

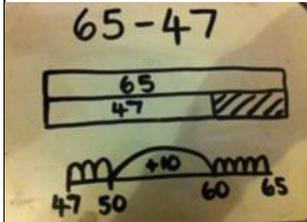
Step C

84-73
Counting up to find the difference



= 11

Firstly counting on to the next ten.

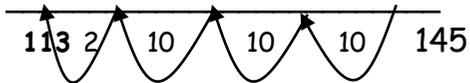


Use Singapore bar method to show difference

Counting back (where appropriate..eg numbers with a small difference, or with

out crossing tens boundaries). Steps back are recorded below the line (or if more appropriate, would be completed mentally, without the number line)

145 - 32 =



solve missing number problems e.g.

27-□=17 □=21-4 10=□-□

Children build on their understanding of subtraction to interpret 14 - 9 as finding the difference between 14 and 9 , or 'How many more must I add to 9 to get 14?'

They may use a number line to count on and find a difference ('Shopkeepers method'), counting on from smallest to largest number. Children count to the next ten first.

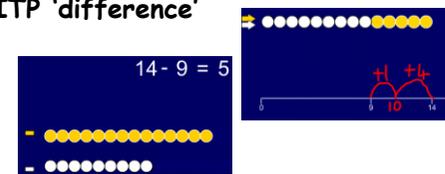
As they become more efficient they will count on using fewer steps.

Children must be able to choose the most efficient method depending on the calculation given, whether to count up or back; whether to perform mentally, or with a number line / jottings

Children develop the relationship between subtraction and addition using 'missing number' problems, supported by number trios.

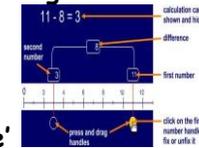
Links are made to work on mental calculation, where children are taught to partition given numbers in a variety of ways, beyond simple tens and ones.

ITP 'difference'



Blank number lines

Number lines showing tens boundaries



ITP 'Number line'

Missing number 'empty box' problems eg 14+ □ =35

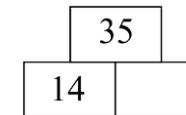
Missing numbers on a number line

Singapore Bar Model

Cuisenaire



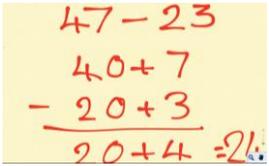
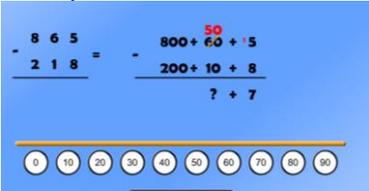
Number trios



Bead strings in tens to 100

as above and also:

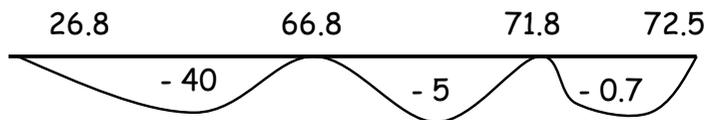
hundreds boundary, method, equation, increase, decrease, inverse, strategy, efficient

	<p>Children learn to partition numbers in a variety of different ways. eg $54 = 50 + 4$, or $40 + 14$</p>		 <p>Straws dienes</p>	
<p>Step D</p>	<p>Expanded written method Children use when competent and confident in using a number line. Partition numbers and then subtract.</p> 	<p>Children set out a subtraction vertically with the largest number on the top of the working out. Begin with numbers that do not require exchanging. This should be taught parallel with children using expanded addition method. Keep the numbers manageable for the individual children. Using the expanded form helps the children to understand the process of exchanging.</p>	<p>Continue to use apparatus such as dienes and place value counters. Money is also useful.</p>  <p>mathsframe column subtraction</p>	<p><i>as above and also:</i></p>
<p>Step E</p>	<p>As children become more confident, they will move onto the compact form of decomposition. Children may need to go back to expanded method if exchanging is not fully understood.</p> <p>There are 754 children in the school. If 286 children go on a school trip, how many will be left in school?</p> $\begin{array}{r} 41 \\ 754 \\ -236 \\ \hline 518 \end{array}$ <p><i>Children do 4-6 and find they do not have enough units to complete the sum so they have to exchange a ten. This means they now have 14 in the units column and only 4 in the tens column.</i></p>	<p>Children will find at this stage that sometimes it is not possible to subtract the number underneath from the number on the top (e.g. you can not do 4-6). Use practical apparatus to demonstrate how to exchange eg dienes or place value counters. Be careful not to move on too quickly at this stage. It is important that children really understand what is actually happening. Children continue to develop mental strategies to subtract, including 'special cases' such as finding $57 - 29$, by subtracting 30, then adding one to adjust, similarly, subtracting 99 mentally or 1.9, using rounding and approximation.</p>	<p>The use of practical apparatus such as dienes and place value counters is extremely important.</p> <p>Dienes base 10 equipment</p> 	<p>units boundary, tenths boundary, strategy, exchange, tens, units, column, place value, place holder, hundredths</p>

Children then move onto using decomposition method for subtraction of numbers involving decimals.

$$72.5 - 45.7 = 16.8$$

$$\begin{array}{r} 67 \cancel{1} 2 . 15 \\ - 45 . 7 \\ \hline 21 . 45 \end{array}$$



Place value counters

Using a number line can also help children to see what is happening to the numbers.

$$£53.94 - £21.78 =$$

Lucy has £53.34 pence in her bank. She spends £29.78. How much money does she have left.

(Use where appropriate)

$$\begin{array}{r} 81 \\ £53.94 \\ -£21.78 \\ \hline £32.16 \end{array}$$

When subtracting decimals, children need to understand that decimal points line up underneath each other.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

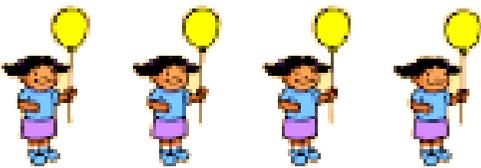
A number line will generally be the best method to use when finding change from notes such as £5 or £20 to avoid lots of subtraction from 0

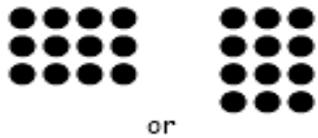
as above and also:

thousandths

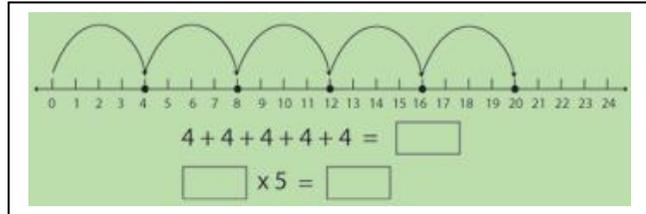
PROGRESSION IN MULTIPLICATION

All children will be encouraged to develop skills in mental calculation alongside development of written methods. Children by the end of Year 6 need to be confident and competent in choosing the most efficient method appropriate to the numbers given. Children are taught to understand multiplication as repeated addition.

Steps	Examples	Suggested activities	Models and images	Vocabulary
FS	<p>Counting in 2's, Number rhymes..One two, buckle my shoe</p> <p>Counting in steps of equal size</p> 	<p>Counting orally in 1's 2's 5's 10's Count repeated groups of the same size</p>	<p>hands socks legs shoes Using equipment, washing line, socks, counting hands, objects related to topic work / classroom environment</p>	<p>count, sort, group, set, choose, collect</p>
Step A	<p>Each child has two legs. How many legs do four children have? How would you say this?</p>  <p style="text-align: center;">$2 + 2 + 2 + 2$</p>	<p>Children draw pictures, or use concrete objects / apparatus Counting on and back in 1's 2's 5's 10's Multiplication taught as repeated addition. Use a number line to show repeated addition in steps of constant size. Use Cuisenaire rods to show equal steps</p>	<p>practical objects related to topic / classroom environment counting in groups number lines (marked intervals) number tracks</p>  <p>eg if the frog hop in twos, how far will he have travelled after 5 hops? Cuisenaire Bead strings and counting stick numicon</p>	<p><u>as above and also:</u></p> <p>How many groups of? ... ks with (sion), double, r double,</p> <p>count out, number sentence, sign, operation</p>
Step B	<p>$3 \times 4 =$ Read as 'three multiplied by four' A chew costs 3p. How much do 4 chews cost?</p>	<p>Drawing an array (3 rows of 4 or 3 columns of 4) gives children an image of the answer. It also helps develop the understanding that 4×3 is the same as 3×4.</p>	<p>ITP Number dial</p>	<p><u>as above and also:</u></p> <p>how many times,</p>



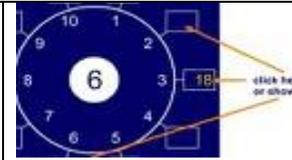
4 x 5: Shown as repeated steps on a number line



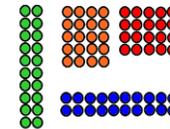
Use a number line to make the link between repeated addition and multiplication, firstly on a marked number line with steps of equal size, then leading to use of a blank number line.

At this stage, children develop memory of multiplication facts to help with mental calculations. Chn are encouraged to use mental recall, and consider the most efficient strategy (eg it wouldn't be appropriate to draw a number line if I know that 4 x 5 is 20)

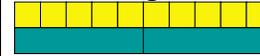
The relationship between multiplication and division is reinforced using arrays and number trios (linked to mental work using number facts and recall of times tables)



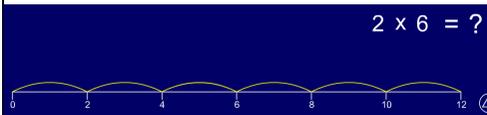
arrays



bead strings



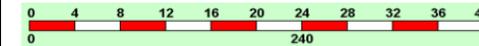
Cuisenaire rods



number lines (marked intervals)

blank number lines

Counting sticks



lots of, groups of, times, multiplied by, multiple of, once, twice, three times, four times, five times, ten times, times as (big, long, wide, and so on), repeated addition, array, row, column, jotting, calculate, calculation, mental calculation, symbol

Step C

There are 16 children in seven classes, how many children all together?

$$16 \times 7 =$$

$$10 \times 7 = 70$$

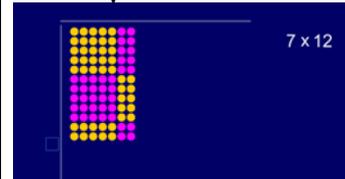
$$6 \times 7 = 42$$

$$70 + 42 = 112$$

An array used to model links with the grid method:

Use the partitioning and recombining methods. Children are encouraged to learn their times tables to help with their calculations. Children use their knowledge of number facts and place value. Chn begin using jottings to support mental work, leading to a grid to record where a more efficient / clearer method is necessary. Chn should be taught to choose

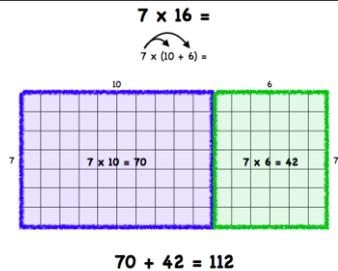
Use arrays to show how numbers can be partitioned



ITP Multi array

Cuisenaire Counting sticks

product, method, equation, divisible by, factor, square number,



16 books were sold. Each book cost £7. How much money was taken?

$16 \times 7 =$

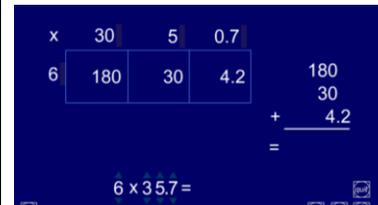
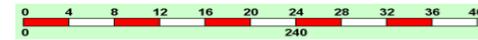
	10	6	
7	70	42	$= 112$

the most efficient methods either mental with jottings, written or calculator. Eg need to understand that to x5, then x10 and half; to multiply by 4- double and double again etc.)

This method can be extended into larger numbers. The partitioning method above is developed leading to the 'grid method'

16 is partitioned into parts (10 and 6) and each of these is multiplied by 7. The two answers are then added together.

This grid method can be extended to include T x HTU and decimal numbers



Use ITP to demonstrate use of grid method for decimal numbers
 Mathsframe



Step E

$72 \times 34 =$
 A cat is 72cm long. A tiger is 34 times longer. How long is the tiger?

	X	70	2	
30	2100	60		
4	280	8		

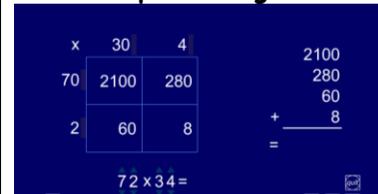
$2100 + 60 = 2160$
 $280 + 8 = 288$
 $2160 + 288 = 2448$

The grid method also works for 'long multiplication'. To use this method, children need a sound knowledge of place value and skills in mental calculation. This method is also used for the multiplication of decimals.

As children become confident with multiplying, introduce the compact method alongside the grid method (end of year 4 or during year 5).

Make sure that the two methods are used side by side to begin with so that children can clearly

ITP Multiplication grid



no new vocabulary

During years 5 and 6, the children will move on to the short compact method to multiply.

$$\begin{array}{r} 73 \\ \times 6 \\ \hline 438 \\ 1 \end{array}$$

Expanded long multiplication:

$$\begin{array}{r} 72 \\ \times 34 \\ \hline 288 \quad (72 \times 4) \\ 2160 \quad (72 \times 30) \\ \hline 2448 \end{array}$$

$$1.6 \times 4$$

$$\begin{array}{r} 1.6 \\ \times 4 \\ \hline 6.4 \\ 2 \end{array}$$

see how the steps are the same but recorded differently.

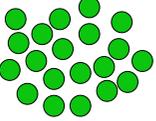
Most children by Year 6 will be using the short compact method to multiply, and written expanded method for long multiplication (where they are multiplying by a 2 digit number)
Children make decisions as to the most efficient methods (may be mental or written, depending on the calculation)

$$\begin{array}{r} 347 \times 4 \\ 300 \quad 40 \quad 7 \\ 4 \boxed{1200} \boxed{160} \boxed{28} \\ \hline 1200 \\ 160 \\ 28 \\ \hline 1388 \end{array}$$

$$\begin{array}{r} 347 \\ \times 4 \\ \hline 1388 \end{array}$$

PROGRESSION IN DIVISION

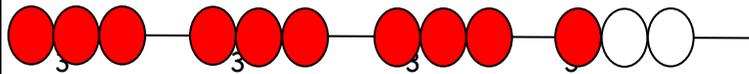
All children will be encouraged to develop skills in mental calculation alongside the development of written methods. Children by the end of Year 6 need to be confident and competent in choosing the most efficient method appropriate to the numbers given.

Steps	Examples	Suggested activities	Models and Images	Vocabulary
FS	<p>Counting in 2's, Number rhymes..One two, buckle my shoe</p> 	<p>Count repeated groups of the same size; Share objects into equal groups and count how many in each group.eg: trays with small compartments for sorting; collections of things: bottle tops, sequins, threads, tiny pieces of fabric, etc. Model sharing out the objects equally. For example: do you all want sequins? I'll put 5 each on your trays. Can you give everybody the same number of these? Have you got the same? Hang up 3 bags outside for making collections. Put a number 2 on each bag. Encourage the children to collect 2 of any treasured object in each bag, for example fir cones or smooth pebbles.</p>	<p>Using equipment, washing line, socks, counting hands, objects related to topic work / classroom environment</p>  	<p>twos, tens, count , count in ones, twos, groups, equal groups, same, set, sort, share, share equally, each, one each, two each....</p>
Step A	<p>Children will experience division as both sharing and grouping, using word problems in context: <u>Sharing:</u> 6 Easter eggs are shared between 2 children. How many eggs do they get each?</p> 	<p>Solve practical problems, using objects in context - grouping and sharing (up to groups of 20). Children will experience equal groups of objects and will count in 2s, 5s and 10s. They will work on practical problem solving activities involving equal sets or groups, Children may also draw pictures as a helpful tool in understanding and solving division problems. They will begin to show steps of equal size on a number line to solve problems</p>	<p>arrays Here are 20 counters. Arrange them in equal rows. Is there a different way to arrange them in equal rows?</p>  <p>ITP grouping ITP' Multiplication',</p>	<p><u>as above, and also</u> inverse, multiply, halving, lots of, groups of, double, half, left, left over</p>

Grouping: eg 6 socks, how many pairs of socks can you make?



Show me on a number line how you could do: $12 \div 3 = 4$

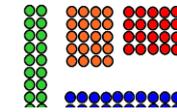


The bead bar will help children with interpreting division calculations such as $12 \div 3$ as 'how many 3s make 12?'

Introduce and begin to use the symbol for division

Use arrays to model the link to multiplication and the inverse (as a means of checking)

Number tracks: eg If the frog hops in 2s, how many hops will there be before he lands on 10?

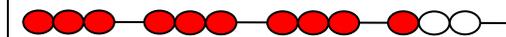


Singapore bar method

Cuisenaire

Number line

Bead strings and bar

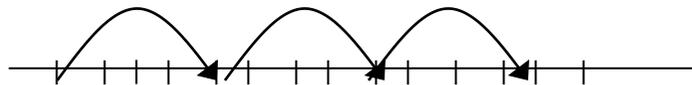
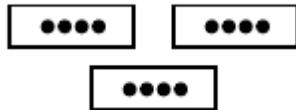


Step B

$12 \div 4$

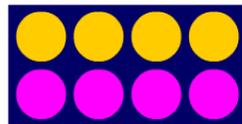
Read as '12 divided into 4 groups' and '12 divided into groups of 4'

There are 13 apples and you can fill each basket with 4 of them. How many baskets will you need



Can you give two multiplication and two division sentences to describe this array?

- $8 \div 2 = 4$
- $8 \div 4 = 2$
- $2 \times 4 = 8$
- $4 \times 2 = 8$



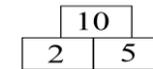
Children will use practical and informal written methods including calculations with remainders. Children use the symbol for division in number sentences, and understand it as both sharing and grouping.

At this stage, children will begin to apply their knowledge of multiplication facts to work out answers mentally... Eg 'I know that $2 \times 3 = 6$. So to share 6 into 2 groups, there will be three in each group'.

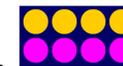
The relationship between multiplication and division is reinforced using arrays, and number trios (linked to mental work using number facts and recall of times tables)

eg 2×4 gives the same answer as 4×2 . They also use the image to show how many twos make 8 and

Number trios



Number lines (intervals of one)



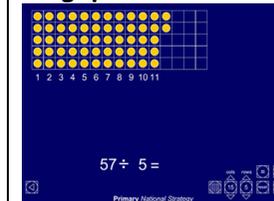
arrays

ITP Multiarray

ITP Grouping

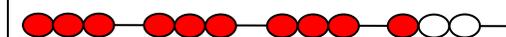
Cuisenaire

Singapore bar method

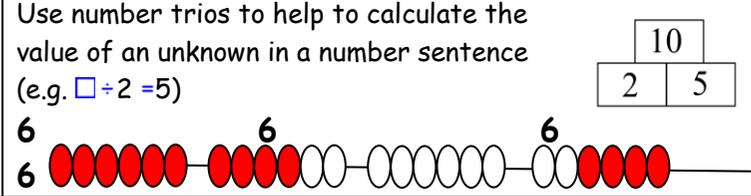
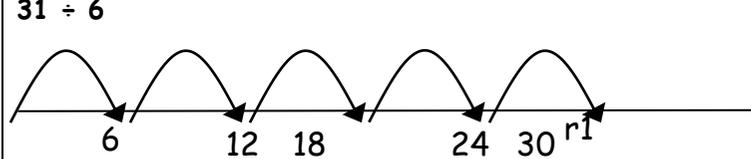
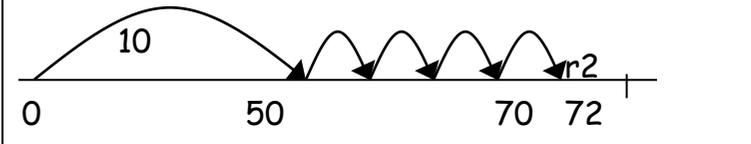
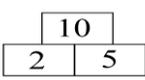
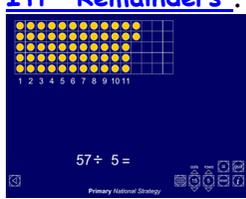


ITP remainders

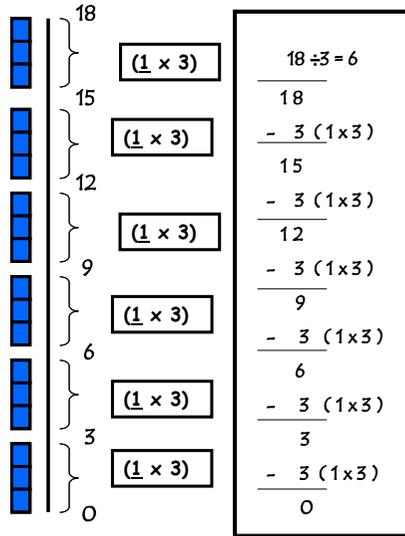
Bead strings and bars



as above and also..
rows and column, remainder, divided by, divided into, array, number trio, related facts

	<p>Use number trios to help to calculate the value of an unknown in a number sentence (e.g. $\square \div 2 = 5$)</p> 	<p>how many fours make 8. Children should use number lines or bead bars to support their understanding.</p>		
<p>Step C</p>	<p>$31 \div 6$</p>  <p>$\square \times 5 = 20 \quad 20 \div \square = 5$</p> <p>$72 \div 5$</p>  <p>$= 14 \text{ r}2$</p>	<p>Show division as repeated steps of constant size and record steps on a number line. Introduce the use of a blank number line to record steps. Include calculations that divide exactly and those that give rise to remainders. Link to knowledge of tables facts, and extend to use place value to find related facts of with multiples of 10. Make more efficient larger jumps of multiples of 10 to divide larger two-digit numbers Understand that division is the inverse of multiplication and vice versa, use related facts to solve missing number problems (number trios)</p> <p>Develop alongside knowledge of mental methods to calculate: Extend mental strategies to divide, using partitioning of numbers. The principles of using a number line can be applied to larger numbers.....recognising that a larger 'jump' or 'chunk' can be made, using known facts in order to be more efficient. Making increasingly efficient jumps</p>	<p>Blank number lines Singapore bar method Cuisenaire Number trios</p>  <p>ITP Multiarray</p> <p>ITP Grouping</p> <p>ITP 'Remainders'</p> 	<p><u>as above and also....</u> sharing and grouping share equally repeated steps equal groups of, divided by, divide, divide into, left over, remainder, factor, quotient, inverse</p>

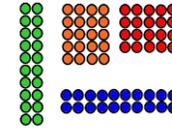
Step D Use childrens knowledge of tables and a vertical number line to introduce 'chunking' or repeated subtraction method.

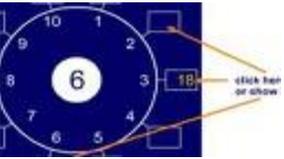
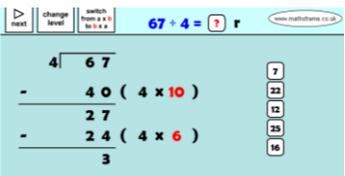
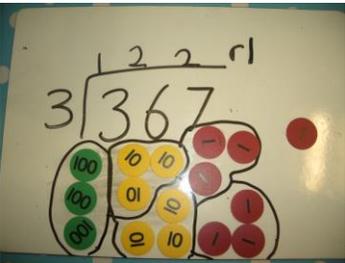


Understand division as repeated subtraction using a vertical line and apparatus to make the links

This method is an important element in understanding division as repeated subtraction.

Use arrays and practical apparatus to demonstrate subtracting the same amount each time.



<p>Step E</p>	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> $\begin{array}{r} 6 \overline{)196} \\ - 60 \quad 6 \times 10 \\ \hline 136 \\ - 60 \quad 6 \times 10 \\ \hline 76 \\ - 60 \quad 6 \times 10 \\ \hline 16 \\ - 12 \quad 6 \times 2 \\ \hline 4 \quad 32 \\ \text{Answer: } \quad 32 \text{ R } 4 \end{array}$ </div> <div style="width: 45%; border: 1px solid black; padding: 5px; text-align: center;"> <p>Repeated subtraction of 'chunks' of the divisor, starting with multiples of 10</p> </div> </div> <div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> $\begin{array}{r} 6 \overline{)196} \\ - 180 \quad 6 \times 30 \\ \hline 16 \\ - 12 \quad 6 \times 2 \\ \hline 4 \quad 32 \\ \text{Answer: } \quad 32 \text{ R } 4 \end{array}$ </div> <div style="width: 45%; border: 1px solid black; padding: 5px; text-align: center;"> <p>Subtraction of larger 'chunks' of the divisor, leading to more efficient calculations with fewer steps.</p> </div> </div> $\begin{array}{r} 24 \overline{)560} \\ 20 - 480 \quad 24 \times 20 \\ \hline 80 \\ 3 \quad 72 \quad 24 \times 3 \\ \hline 8 \\ \text{Answer: } 23 \text{ R } 8 \end{array}$	<p>The children need to see that as the numbers get larger, larger chunks are more efficient. Multiples of the divisor (large chunks) is taken away. Good recall of multiplication facts, and mental partitioning of numbers are needed to see the size of the 'chunk' The children repeatedly subtract 'chunks' of numbers..... the most efficient building on largest chunks, using mental methods to support. Children develop mental methods using multiples of 10 to make more efficient chunks as in the bottom example. It can help to write down relevant facts at the side of the calculation eg $10 \times 6 = 60$, $20 \times 6 = 120$, $30 \times 6 = 180$ etc Children need to be fluent in use of tables to be able to use chunks effectively.</p>	<p>Singapore bar method</p> <p>ITP: Number Dial</p>   <p>mathsframe - division by chunking</p>	<p><u>as above and also...</u> divisible by share equally repeated steps equal groups of, divided by, divide, divide into, left over, remainder, factor, quotient, inverse</p>
<p>Step F</p>	<p>Short 'algorithm' or 'bus stop' method</p> <p>Use an expanded form to begin with which will link with the chunking method already known.</p> $\begin{array}{r} 62 \\ 6 \overline{)252} \\ \underline{240} \\ 12 \end{array}$ <p>Encourage the children to use tables facts and to refer to correct place value eg to divide 250 by 6 rather than how many sixes in 25.</p>	<p>During years 5 and 6 the more traditional algorithm or 'bus stop' method should be introduced. Children have often found this method difficult in the past due to not understanding the actual process and it is important that the shorter method is not introduced too early.</p>	<p>The use of practical apparatus and models and images is crucial in embedding children's understanding of this method.</p> 	<p>no new vocabulary</p>

This method can be extended to division by 2 digit numbers.

$$\begin{array}{r} 23 \text{ r } 10 \\ 24 \overline{)562} \\ \underline{480} \\ 82 \\ \underline{72} \\ 10 \end{array}$$

The children will then move on to the more compact version when they are confident with the process.

$$\begin{array}{r} 97 \\ 3 \overline{)2921} \end{array}$$

mathsframe - formal written division



The screenshot shows a digital interface for formal written division. The title is "Long Division - Formal Written Method". The main display shows the division of 344 by 16, resulting in a quotient of 21 and a remainder of 8, written as $21 \frac{1}{2}$. The steps are shown as follows:

$$\begin{array}{r} 21 \text{ r } 8 = 21 \frac{1}{2} \\ 16 \overline{)344} \\ \underline{320} \quad 16 \times 20 \\ \underline{024} \quad 16 \times 1 \\ \underline{008} \end{array}$$

By the end of Year 6 the children are expected to be using 'traditional' written methods for all 4 main operations. It is important however that these are not introduced too soon and that practical apparatus and images be used to embed the children's understanding of what they are doing at each stage.